Eighth Annual Ottawa-Carleton Applied Analysis Day

University of Ottawa, October 10, 2025

This annual one-day workshop brings together researchers from Carleton University and the University of Ottawa working in applied analysis and related fields. It features two keynote talks, given this year by Prof. Lia Bronsard (McMaster University) and Prof. Jean-Philippe Lessard (McGill University), as well as a collection of short talks by students and professors from the Ottawa area and beyond.

Schedule

All the talks will take place in STM 464 (150 Louis-Pasteur Private, Ottawa, ON K1N 9B4).

Time	**Speaker**
08:30am-09:00am	Welcome
09:00am-10:00am	**Jean-Philippe Lessard**, McGill University *The Path from Lagrange's Triangle to the Figure Eight: Resolving Marchal's Conjecture*
10:00am-10:30am	Coffee break
10:30am-11:00am	**Elise Woodward**, University of Ottawa *Don't Let the Budworms Bite: Patch-Based Optimal Control of Spruce Budworm Dynamics*
11:00am-11:30am	**Cristina Stoica**, Wilfrid Laurier University *The Metriplectic Kepler Problem*
11:30am-12:00pm	**Archishman Saha**, University of Ottawa *Deterministic Behaviour in Stochastic Collective Hamiltonian Systems*
12:00pm-01:30pm	Lunch break
01:30pm-02:30pm	**Lia Bronsard**, McMaster University *Nonlocal Isoperimetric Problems: Lamellar Pattern, Lens Cluster, and a New Partitioning Problem*
02:30pm-03:00pm	**Zhiyi Lin**, University of Ottawa *Quantum Optimal Transport and Barycenters of Quantum States*
03:00pm-03:30pm	Coffee break
03:30pm-04:00pm	**Giusy Mazzone**, Queen's University *Periodic Motions of a Harmonic Oscillator in a Newtonian Fluid*
04:00pm-04:30pm	**Mohamed Barakat**, University of Ottawa *On the Convergence of Nonlinear Reduced Basis Methods*
04:30pm-05:00pm	**Yves Bourgault**, University of Ottawa *Linearly-Implicit Backward Difference Formulas for Navier–Stokes Equations*

Abstracts

The Path from Lagrange's Triangle to the Figure Eight: Resolving Marchal's Conjecture
Jean-Philippe Lessard, McGill University

The three-body problem continues to surprise us with its blend of symmetry, geometry, and dynamics. Among its most celebrated solutions are Lagrange's equilateral triangle orbit from the 18th century and the striking figure-eight choreography discovered by Moore in 1993 and later established rigorously by Chenciner and Montgomery. In this talk, I will show that these two iconic solutions are connected through the P12 family of periodic orbits introduced by Marchal, thereby settling his long-standing conjecture from 1999. Our proof is computer-assisted: we combine analytic arguments, symmetries, a reformulation of the problem as a delay differential equation, fixed-point methods, the Newton–Kantorovich theorem, Fourier analysis, and interval arithmetic to rigorously control the continuation path between the triangle and the eight. This provides the first complete and rigorous proof of Marchal's conjecture.

Don't Let the Budworms Bite: Patch-Based Optimal Control of Spruce Budworm Dynamics
Elise Woodward, University of Ottawa

Spruce Budworm (SBW) are one of the most expensive Canadian pests. These insects defoliate Firs and Spruce trees, devastating boreal forests, leading to losses in timber harvesting and increased climate effects. One commonly implemented method of control is aerial pesticide spraying. In this talk, I formulate a spatially explicit semi-discrete SBW- inspired model and apply controls. On each patch, population dynamics are modelled by continuous differential equations. A set of difference equation models dispersal and reproduction. The control is applied as a separate impulse. I analyze the optimal control intensity in a finite time horizon relying on Pontryagin's Maximum Principle and the Forward-Backward Sweep Algorithm. I discuss the implications of the control schedule on SBW populations and the future directions of the research.

The Metriplectic Kepler Problem
Cristina Stoica, Wilfrid Laurier University

We construct a metriplectic (energy-preserving, entropy-producing) deformation of the bound Kepler dynamics that monotonically tends to the circular trajectories while keeping the energy fixed. Two equivalent formulations are presented: (i) on canonical phase space and (ii) on the space of $\mathfrak{so}(4)$ invariants.

Deterministic Behaviour in Stochastic Collective Hamiltonian Systems
Archishman Saha, University of Ottawa

We consider stochastic perturbations of Hamiltonian systems by noise arising from collective Hamiltonians. We show that these perturbations typically preserve many symmetry-related features of the deterministic system even though the stochastic differential equations governing the dynamics are not symmetric in general. When the deterministic Hamiltonian is symmetric under a free, proper and canonical Lie group action, we show that the projection of a solution of the stochastic system onto the reduced space evolves deterministically.

Nonlocal isoperimetric problems: lamellar pattern, lens cluster, and a new partitioning problem
Lia Bronsard, McMaster University

We first present a nonlocal isoperimetric problem for three interacting phase domains, related to the Nakazawa-Ohta ternary inhibitory system which describes domain morphologies in a triblock copolymer. We consider global minimizers on the two-dimensional torus, in the droplet regime where some of the species have vanishingly small mass but the interaction strength is correspondingly large. In this limit there is splitting of the masses, and each vanishing component rescales to a minimizer of an isoperimetric problem for clusters in 2D. These results have led to a new type of partitioning problem that I will also describe. These represent work with S. Alama, X. Lu, C. Wang, S. Vriend and M. Novack.

Quantum Optimal Transport and Barycenters of Quantum States
Zhiyi Lin, University of Ottawa

We discuss the notion of barycenters (or Fréchet means) in the framework of Quantum Optimal Transport (QOT), extending the concept of Wasserstein barycenters from classical probability measures to the noncommutative setting of quantum states. After introducing the quantum Wasserstein distance and its variational formulation, we establish the existence of quantum barycenters by employing the direct method in the Calculus of Variations, as well as duality results. Finally, we present explicit examples in the case where the barycenter (Fréchet mean) is taken among Gaussian quantum states, where closed-form expressions can be derived and the geometric features of the quantum Wasserstein space can be visualized.

Periodic Motions of a Harmonic Oscillator in a Newtonian Fluid
Giusy Mazzone, Queen's University

We consider the periodic motions of a harmonic oscillator (consisting of a mass-spring system) and a fluid occupying an infinite three-dimensional channel. The fluid flow is governed by the Navier–Stokes equations subject to a prescribed time-periodic flow rate. External time-periodic body forces (with the same period as the flow rate) may also be applied on the fluid and on the mass. Physical intuition might suggest that, under a prescribed time-periodic flow rate and time-periodic forces, the fluid could exert on the oscillator a time-periodic force having frequency matching the natural frequency of the oscillator. In this scenario, the phenomenon of resonance would occur (since the oscillator is undamped), and the generic motion of the oscillator would be characterized by oscillations with increasing amplitude. In mathematical terms, this means that no periodic motion would exist. We show that this intuition is not correct and, in fact, the fluid dissipation provides sufficient damping to guarantee the existence of periodic motions for the fluid-oscillator system, no matter what the period of the flow rate is, if the flow rate is "small". In addition, the fluid flow tends to the "generalized time-periodic Poiseuille flow" at channel inlets/outlets.

On the Convergence of Nonlinear Reduced Basis Methods
Mohamed Barakat, University of Ottawa

The Reduced Basis (RB) method is a powerful computational framework for the efficient evaluation of functional outputs associated with parametric partial differential equations (PDEs). The method constructs a reduced linear approximation space by spanning pre-computed high-fidelity solution snapshots obtained at optimally selected parameter values, typically through a greedy algorithm. If the Kolmogorov N-width of the solution manifold decays rapidly, the linear RB method can deliver high accuracy with only a small number of snapshots. Moreover, under the assumption of affine parameter dependence, both the RB approximation and certified error bounds can be evaluated efficiently via the classical offline–online decomposition.

In scenarios where the Kolmogorov N-width decays slowly, such as in transport-dominated problems, high-dimensional RB spaces are required to reach a prescribed accuracy. This, in turn, leads to significant computational overhead in the online phase, diminishing the efficiency of the approach. These challenges motivate the development of nonlinear model reduction techniques such as nonlinear RB methods. While the concept of nonlinear model reduction has been investigated in the literature, existing approaches often lack rigor, generality, or completeness.

In this work, we propose a novel nonlinear RB algorithm based on a systematic partitioning of the parameter domain. We provide a rigorous theoretical analysis that characterizes the convergence of the method and, in particular, establish bounds on the number of parameter subdomains required to attain a prescribed accuracy.

Linearly-Implicit Backward Difference Formulas for Navier-Stokes Equations

Yves Bourgault, University of Ottawa

We propose a linearly-implicit method (called LBDFT) to solve the incompressible Navier-Stokes equations. Linearly-implicit methods have an algorithmic complexity that lies between fully-implicit and semi-implicit time-stepping schemes. In LBDFT, the nonlinear advection in the Navier-Stokes equations is split into three linear terms using a Taylor series expansion. One term is taken explicitly and the other two are updated with the linear diffusion term at each time step. Linearly-implicit methods were also proposed in [Garcia-Archilla & Novo, IMA J Num Analysis, 2022][Wang et als, CAM, 2023], in this case based on extrapolation formulae as for semi-implicit methods. These methods are then compared with various fully-implicit and semi-implicit time-stepping methods in terms of accuracy, stability, computing time and ability to compute various flows. We first use two standard test cases to assess the methods. We observed that linearly-implicit methods are more CPU efficient compared to fully-implicit BDF, both at second- and third-order of accuracy. Our third test case explores the ability of the methods to compute steady flows at high Reynolds numbers. LBDFT was able to compute steady cavity flows for Reynolds up to 500,000. Our last test case explores unsteady flows at large Reynolds numbers. It was observed that the linearly-implicit methods allow significantly larger critical time step (40-50 times larger) compared to the semi-implicit methods, the latters needing a stabilization term to maintain their stability. This article is co-authored with Kak Choon Loy, Faculty of Computer Science and Mathematics, Universiti Malaysia Terengganu, Malaysia.
